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1

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Non-Newtonian Gravitational Forces and the Greenland Ice-Sheet Experiment

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Abstract

The results of an experiment to test Newton's Inverse-Square Law of Gravitation in the Greenland ice-cap were announced recently. The anomalous gravity gradient which was found can be explained either by an unrecognized anomaly in the density of the rocks under the ice sheet, or by the existence of a non-Newtonian component of the gravitational force. Here we focus on the latter possibility, and find that the force would be attractive, with a strength between about 2.4% and 3.5% that of Newtonian gravity, and a range between about 225 m and 5.4 km.

The experimental technique adopted by the Greenland collaboration[1]) originated with G. B. Airy in the 19th century[2]), and has been modified to test the inverse square law. The objective is to measure the change of the acceleration due to gravity, g, with depth, z, in the earth, and compare this with the prediction based on Newtonian gravity and knowledge of the density of material between the surface and depth z. In a simple, spherically-layered model earth we would have,

$$\Delta g(z)_{\mathrm{Newton}} \equiv g(z) - g(0) = z\left[\gamma - 4\pi G_{\mathrm{lab}}\rho\right] \quad , \tag{1}$$

where we have assumed that $z << R_{\oplus}$, the radius of the earth. Here, γ is the "free-air gradient," the variation of gravity with depth, ignoring the gravitational effect of the surrounding material. In our simple model earth

$$\gamma = (308.3 + 0.5\cos^2\phi)\mu\mathrm{Gal}\, m^{-1} \quad , \tag{2}$$

corresponding to an increase of gravity with depth (1 Gal $= 10^{-2}ms^{-2}$). (In eq. 2 ϕ is the latitude). G_{lab} is the value of Newton's constant measured in laboratory experiments, and ρ is the density of the material through which the gravity measurements are taken. The second term in eq. (1) is the "double Bouguer gradient." It represents the gravitational attraction of the surrounding material, and causes gravity to decrease with depth. For ice, $\rho \approx 0.91gcm^{-3}$, so that

$$4\pi G_{\mathrm{lab}}\rho \simeq 80\mu\mathrm{Gal}\, m^{-1} \quad . \tag{3}$$

After developing the model value, $\Delta g(z)_{\mathrm{Newton}}$, the gravity anomaly

$$\Delta g(z)_{\mathrm{anomaly}} = \Delta g(z)_{\mathrm{measured}} - \Delta g(z)_{\mathrm{Newton}} \tag{4}$$

is formed. The non-zero anomaly found in Greenland[1]) then represents some unrecognized anomaly in the Newtonian model. This anomaly could be either Newtonian in nature, where attention focuses on the free-air gradient, or the effect of some non-Newtonian gravitational force. The experimental group has addressed carefully the former issue, where they have succeeded in quantifying the effect on $\Delta g_{\mathrm{Newton}}$, of unknown density anomalies in the bedrock below the ice-sheet.[1]) In our simple model (1), these density anomalies would show up as local or regional structure in γ.

The conclusion of the collaboration,[1]) which represents a major improvement upon earlier Airy experiments, is that a relatively large and geologically unexpected amount of excess density of $0.3gcm^{-3}$ would be required. The technique used to reach this conclusion, ideal body analysis,[9] will allow the development of potentially much more definitive experiments of this type, where density anomalies can be made even more unlikely. Here we will focus on the second possible explanation for the non-zero gravity anomalies found by the Greenland collaboration, namely, non-Newtonian gravitational forces. To this end a non-Newtonian expression for $\Delta g_{\text{anomaly}}$ must be developed.

We start from a simple Yukawa potential addition to the Newtonian gravitational potential at distance r from a point mass, m

$$V_{NN}(r) = -\frac{G_\infty m}{r}\left[1 + \alpha\, e^{-r/\lambda}\right] \quad , \tag{5}$$

where "NN" denotes non-Newtonian.

Here α is a strength parameter, and λ is the range of the non-Newtonian force. Such modifications of Newtonian gravity occur in modern quantum gravity theories.[10]

In equation (5) G_∞ is the (true value of) Newton's constant at large separation ($r >> \lambda$), while the laboratory value (valid for small separations),

$$G_{\text{lab}} = (1 + \alpha)G_\infty \quad , \tag{6}$$

must be used in Δg_{Newton}. Our task now is to obtain a non-Newtonian model value for the gravity anomaly

$$\Delta g(z)_{\substack{\text{model} \\ \text{anomaly}}} = \Delta g(z)_{NN} - \Delta g(z)_{\text{Newton}} \quad , \tag{7}$$

where Δg_{NN} is obtained from eq. (5) by integration over the earth.

The Newtonian component of eq. (5) is easily dealt with, while the contribution of the Yukawa term can be determined from the following two formulae, which give the Yukawa contribution, δg, to the gravitational acceleration at height z above a uniform sphere of density ρ, radius R,

$$\delta g(z) = 4\pi G_\infty \rho\, \alpha\, \lambda \left\{R\cosh\left(\frac{R}{\lambda}\right) - \lambda \sinh\left(\frac{R}{\lambda}\right)\right\} \frac{e^{-(R+z)/\lambda}}{R+z}\left(1 + \frac{\lambda}{R+z}\right) \quad , \tag{8}$$

and at depth z below the surface,

$$\delta g(z) = 4\pi G_\infty \rho\, \alpha\, \lambda \frac{(R+\lambda)e^{-R/\lambda}}{R-z}\left\{\cosh\left(\frac{R-z}{\lambda}\right) - \frac{\lambda}{R-z}\sinh\left(\frac{R-z}{\lambda}\right)\right\} \tag{9}$$

To apply these results to the Greenland experiment, we approximate the earth as spherically layered ($\rho \equiv \rho(r)$), with a layer of ice of thickness $h \approx 2km$ of density $\rho_I \approx 0.91gcm^{-3}$, overlying rocks of density $\rho_R \approx 2.7gcm^{-3}$. Under the assumption that λ is much less than the thickness of the earth's crust we may replace all the rocks below the ice by a uniform sphere of density ρ_R and radius $(R_\oplus - h)$ as far as the Yukawa term is concerned. The Yukawa contribution of the spherical shell of ice is then simply the difference between that of uniform spheres of ice of radius $R_\oplus$ and $(R_\oplus - h)$.

The non-Newtonian value of Δg is then,

$$\begin{aligned} \Delta g(z)_{NN} &= 4\pi G_\infty \left[(R_\oplus - z)^{-2} \int_0^{R_\oplus - z} dr' r'^2 \rho(r') - R_\oplus^{-2} \int_0^{R_\oplus} dr' r'^2 \rho(r') \right] \\ &+ 2\pi G_\infty \alpha\lambda \left\{ (\rho_R - \rho_I) \left[e^{-(h-z)/\lambda} - e^{-h/\lambda} \right] + \rho_I \left[e^{-z/\lambda} - 1 \right] \right\}, \quad (10) \end{aligned}$$

where the first term is the contribution of the Newtonian component in eq. (5), and we have assumed $\lambda << R_\oplus$, and $0 < z < h$, in the second term.

The Newtonian model value of Δg is

$$\Delta g(z)_{\mathrm{Newton}} = 4\pi G_\infty (1 + \alpha) \left[(R_\oplus - z)^{-2} \int_0^{R_\oplus - z} dr' r'^2 \rho(r') - R_\oplus^{-2} \int_0^{R_\oplus} dr' r'^2 \rho(r') \right] \quad (11)$$

where we have used eq. (6). So, from eqs. 7, 10 and 11 we obtain the non-Newtonian form of the anomaly,

$$\Delta g(z)_{\substack{\mathrm{model} \\ \mathrm{anomaly}}} = 4\pi G_\infty \alpha \left\{ \rho_I z + \frac{\lambda}{2} \left[(\rho_R - \rho_I) e^{-h/\lambda} \left(e^{z/\lambda} - 1 \right) - \rho_I \left(1 - e^{-z/\lambda} \right) \right] \right\} \quad (12)$$

which is to be compared with the experimental values, eq. (4).

The anomalous gravity gradient is

$$\frac{d}{dz} \Delta g_{\substack{\mathrm{model} \\ \mathrm{anomaly}}} = 4\pi G_\infty \alpha \left\{ \rho_I + \frac{1}{2} \left[(\rho_R - \rho_I) e^{(z-h)/\lambda} - \rho_I e^{-z/\lambda} \right] \right\} . \quad (13)$$

Using

$$G_{\mathrm{lab}} \simeq 6.67 \mu Gal m^{-1} / gcm^{-3} , \quad (14)$$

we find that the gradient in two limiting cases is,

$$\frac{d}{dz}\Delta g_{\substack{\text{model}\\\text{anomaly}}} \approx \begin{cases} 2\pi G_{\text{lab}}\rho_I\alpha \approx 40\alpha\mu Gal m^{-1}, z\approx 0 \\ 2\pi G_{\text{lab}}(\rho_I+\rho_R)\alpha \approx 160\alpha\mu Gal m^{-1}, z=h \end{cases} \text{for } \lambda << h \quad (15)$$

and

$$\frac{d}{dz}\Delta g_{\substack{\text{model}\\\text{anomaly}}} \approx 2\pi G_{\text{lab}}\rho_R\alpha \approx 120\alpha\mu\text{Gal}m^{-1}, 0<z<h \text{ for } \lambda >> h \ . \quad (16)$$

Anomalous gradients interpolate smoothly between these values for intermediate λ.

It is immediately clear from comparing these last two formulae with the experimental results that λ is poorly constrained, while α must be 2-3% (attractive) in size. A more quantitative statement of these features is contained in our paper[4]. The best fit corresponds to α = 3.1% and λ = 805m with χ^2 = 0.14 per degree of freedom. The minimum range solution with χ^2 = 1 per degree of freedom has α = 3.5% and λ = 225m, which is consistent with the "Rapp bound,"[5] as derived by Stacey *et al.*[6]

$$|\alpha\lambda| < 14m \ . \quad (17)$$

The maximum range solution has α = 2.4% and λ = 5.4 km. This definitely exceeds the Rapp bound, eq. 17), but the bound itself may be too low by a significant numerical factor[6].

We find, then, that a non-Newtonian explanation of the Greenland results, with one Yukawa component, requires an attractive force with parameters which are consistent with, but tending to be larger than those needed to explain the results of Eckhardt *et al.*[7] For a single Yukawa force the Greenland results are inconsistent with those of Stacey *et al.*[6,8]

The results of Eckhardt *et al.* have been reconciled[7,9] with those of Stacey *et al.*[6,8] by using two Yukawa components, which are suggested by our quantum gravity phenomenology.[10] However, in our opinion it is premature to regard such a result as supporting the two-component phenomenology, when the existence of any non-Newtonian phenomenon has yet to be definitively established. Indeed, Parker has reported[11] that much smaller density contrast anomalies than needed in Greenland can provide Newtonian explanations for the Australian-mineshaft and North-Carolina-tower gravity anomalies. Therefore, although the Greenland results could be reconciled with those of Stacey *et al.* in the two-component picture, it is not necessary to pursue that point here.

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